Exercise 41

(a) In mechanics, the ***moment M of a force F about a point*** O is defined to be the magnitude of **F** times the perpendicular distance d from O to the line of action of **F**. The ***vector moment*** **M** is the vector of magnitude M whose direction is perpendicular to the plane of O and **F**, determined by the right-hand rule. Show that $\mathbf{M} = \mathbf{R} \times \mathbf{F}$, where **R** is any vector from O to the line of action of **F**. (See Figure 1.3.10.)

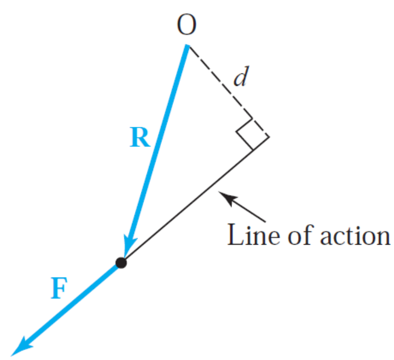


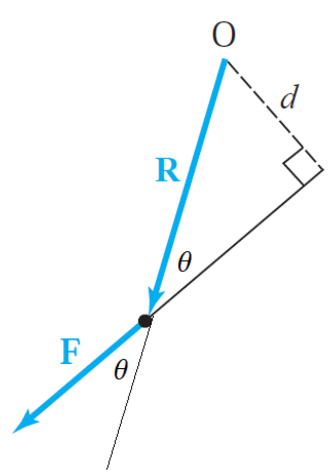
figure **1.3.10** Moment of a force.

(b) Find the moment of the force vector $\mathbf{F} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ newtons about the origin if the line of action is $x = 1 + t$, $y = 1 - t$, $z = 2t$.

Solution

Part (a)

Let θ be the angle between **R** and **F**.



From the figure,

$$\sin\theta = \frac{d}{\|\mathbf{R}\|},$$

which means the perpendicular distance is

$$d = \|\mathbf{R}\| \sin\theta.$$

According to the definition, the moment M is the magnitude of $\mathbf{F}$ times d.

$$M = \|\mathbf{F}\|d = \|\mathbf{F}\|\|\mathbf{R}\| \sin\theta$$

The right side is how the magnitude of the cross product is defined.

$$M = \|\mathbf{F} \times \mathbf{R}\| = \|\mathbf{R} \times \mathbf{F}\|$$

Since the vector moment's direction is perpendicular to the plane containing $\mathbf{R}$ and $\mathbf{F}$, the cross product of $\mathbf{R}$ and $\mathbf{F}$ will give the correct direction.

$$\mathbf{M} = \pm(\mathbf{R} \times \mathbf{F})$$

To follow the right-hand corkscrew rule, the positive sign is chosen.

$$\mathbf{M} = \mathbf{R} \times \mathbf{F}$$

Part (a)

The moment M is the magnitude of $\mathbf{F}$ times the perpendicular distance from the origin to the line of action. The magnitude of $\mathbf{F} = (1, -1, 2)$ is

$$\|\mathbf{F}\| = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{6}.$$

The distance from the origin to the line of action ($x = 1 + t$, $y = 1 - t$, $z = 2t$) is

$$d(t) = \sqrt{[(1 + t) - 0]^2 + [(1 - t) - 0]^2 + [(2t) - 0]^2} = \sqrt{6t^2 + 2}.$$

The minimum of $d(t)$ is the perpendicular distance: $d_\perp = \sqrt{2}$. Therefore,

$$M = \|\mathbf{F}\|d_\perp = \sqrt{6}\sqrt{2} = \sqrt{12}.$$